Practice Problems: Alternating Series and Absolute Convergence

These practice problems supplement the example and exercise videos, and are typical exam-style problems. Some problems may be considered more involved or time-consuming than would be appropriate for an exam - such problems are noted.

Concept: Understand when the alternating series test can be applied and how it works.

TRUE or FALSE: For the following questions, answer TRUE or FALSE. If you answer TRUE, explain why the statement is always true, and if you answer FALSE, provide an example in which the statement is not true. (NOTE: on exams, most of the points on a true-false question are based on the explanation or the counter-example).

1. The alternating series test is used when a series has terms which alternate between positive and negative, are decreasing in magnitude, and have limit 0 as \( n \to \infty \).

2. If the first several terms are all positive, the alternating series test cannot be used.

3. If an alternating series converges, its limit must fall between any two subsequent partial sums, or equal one of the previous two partial sums.

4. It is impossible to prove that a series diverges with the alternating series test.

5. \(-1^n\) is the same as \((-1)^n\).

Skill: Use the alternating series test to show series convergence, or to find that some other test must be used.

6. Use the alternating series test to attempt to show the following series converge, or state why the test cannot be used on the series.

(a) \(\sum_{n=1}^{\infty} \frac{(-n)^n}{n^2 + n + 1}\)

(b) \(\sum_{n=1}^{\infty} \frac{(-1)^n e^n}{n!}\)

(c) \(\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n + 1)}{(n + 5)}\)

(d) \(\sum_{n=1}^{\infty} \frac{(-n)^2}{2^n}\)

(e) \(\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln n}\)

7. Give lower and upper bounds for the values of the following alternating series by calculating six terms and finding partial sums:

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \)

(c) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{2^n} \)

Concept: Understand the difference between conditional and absolute convergence.

TRUE or FALSE: For the following questions, answer TRUE or FALSE. If you answer TRUE, explain why the statement is always true, and if you answer FALSE, provide an example in which the statement is not true. (NOTE: on exams, most of the points on a true-false question are based on the explanation or the counter-example).

8. Every series that is absolutely convergent is also conditionally convergent.

9. Every series that is absolutely or conditionally convergent is also convergent.

10. If a series has all positive terms, then it MUST be either absolutely convergent or divergent.

Skill: Show that a series is convergent by showing that it is absolutely convergent.

11. Attempt to show that the following series are convergent:

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^{n^2-n+1}}{n^2 - n + 1} \)

(b) \( \sum_{n=1}^{\infty} \frac{\sin^3 n}{n \sqrt{n}} \)

Skill: Determine whether a series is absolutely or conditionally convergent.

12. Determine whether the following series are absolutely convergent, conditionally convergent, or divergent:

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}} \)

(b) \( \sum_{n=1}^{\infty} \frac{(-2)^n}{3^n + 1} \)